18.06 Professor Edelman Quiz 1 October 3, 2012

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1	T 9	2-132	Andrey Grinshpun	2-349	3-7578	`agrinshp`
2	T 10	2-132	Rosalie Belanger-Rioux	2-331	3-5029	`robr`
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4	T 11	2-132	Rosalie Belanger-Rioux	2-331	3-5029	`robr`
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6	T 1	2-132	Tiankai Liu	2-491	3-4091	`tiankai`
7	T 2	2-132	Tiankai Liu	2-491	3-4091	`tiankai`

1 (22 pts.)

Let $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 4 \end{pmatrix}$ and $M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 3 & 4 \end{pmatrix}$.

a) (5 pts.) Which are the pivot columns and which are the free columns of A ? Why?.

b) (5 pts.) Which are the pivot columns and which are the free columns of M ? Why?

c) (6 pts.) For which $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ are there solutions to $Ax = b$? For those b, write down the complete solution.

d) (6 pts.) For which $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ are there solutions to $Mx = b$? For those b, write down the complete solution.

2 (24 pts.)

Consider the vector space of polynomials of the form $p(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d can be any real numbers. Are the following subspaces? Explain briefly in a way that we are sure you understand subspaces.

a) (6 pts.) Those $p(x)$ for which $p(1) = 0$.

b) (6 pts.) Those $p(x)$ for which $p(0) = 1$.

c) (6 pts.) Those $p(x)$ for which $a + b = c + d$.

d) (6 pts.) Those $p(x)$ for which $a^2 + b^2 = c^2 + d^2$.

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3 (27 pts.)

a) (9 pts.) Find an LU decomposition of the matrix $A = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$, where we assume $a \neq 0$. L is unit lower triangular (1's on the diagonal) and U is upper triangular.

b) (9 pts.) Find a "PU" decomposition of the matrix $A = \begin{pmatrix} 0 & a & b \\ c & d & e \\ 0 & 0 & f \end{pmatrix}$, where P is a permutation matrix, and U is upper triangular.

c) (9 pts.) Find an "X'X" decomposition of the matrix $A = \begin{pmatrix} a^2+b^2+c^2 & ad+be+cf \\ ad+be+cf & d^2+e^2+f^2 \end{pmatrix}$.
The matrix X that you need to find satisfies $A = X^T X$, and need not be a square matrix.

4 (27 pts.)

Either construct a matrix A or argue that it is impossible, where the nullspace of A is exactly the multiples of $(1, 1, 1, 1)$ and the dimensions (number of rows, number of columns) of A are

a) (9 pts.) 2×4

b) (9 pts.) 3×4

c) (9 pts.) 4×4

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